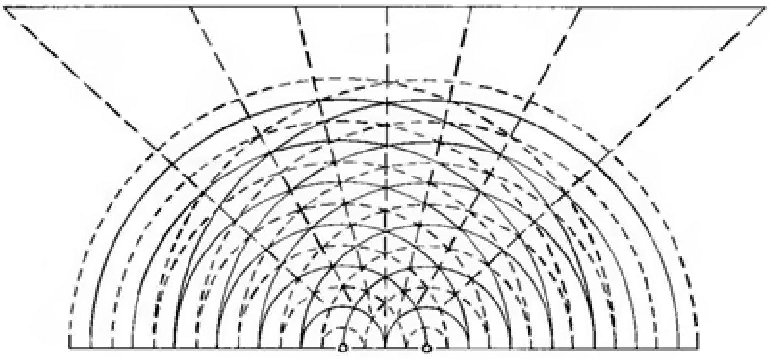

Second Edition

Introduction to Optics

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PHYSICAL CONSTANTS

Speed of light	$c = 2.998 \times 10^{8}$ m/s
Electron charge	$e = 1.602 \times 10^{-19}$ C
Electron rest mass	$m = 9.109 \times 10^{-31}$ kg
Planck constant	$h = 6.626 \times 10^{-34}$ Js
Boltzmann constant	$k = 1.3805 \times 10^{-23}$ J/K
Permittivity of vacuum	$\epsilon_0 = 8.854 \times 10^{-12}$ C^2/N-m^2
Permeability of vacuum	$\mu_0 = 4\pi \times 10^{-7}$ T-m/A

Preface

Optics is today perhaps the most flourishing area of both theoretical and applied physics. Since the 1960s the parallel emergence and development of lasers, fiber optics, and a variety of semiconductor sources and detectors have revitalized the field. The need for a variety of updated optics texts with different approaches and emphases is therefore apparent, both for the student of optics and for the laborer in the field who needs an occasional review of the basics.

With *Introduction to Optics* we propose to teach introductory modern optics at an intermediate level. Except for several of the final chapters (19, 20, 22, 25, 27), which are written at a somewhat higher level, the text assumes as background a good course in introductory physics, at the level usually given to physics and engineering majors, and at least two semesters of calculus. The book is written at the level of understanding appropriate to the average sophomore physics major, who has usually completed the necessary physics and mathematics prerequisites as a freshman. Encompassing the traditional areas of college optics, as well as several rather new ones, the text can be designed for either a half- or a full-year course. We believe that the vitality and importance of optics today warrant readjustment of curricula to provide for a full year of optics early in the program.

For those who are familiar with the first edition, it may be helpful to summarize the major changes introduced in this second edition. Two entirely new chapters dealing with laser-beam characteristics and nonlinear optics have been added. The new laser chapter now appears, together with the two earlier laser chapters, toward the end of the book, where the three function as a unit. In addition, the chapter on fiber optics has been greatly expanded and moved to a later chapter. Several new sections have been introduced. They are Ray Tracing and The Thick Lens (Chapter 4), Doppler Effect (Chapter 8), and Evanescent Waves (Chapter 20). Worked examples are now highlighted within the text, and 175 new problems have been added to the chapter exercises.

Specific features of the text, in terms of coverage beyond the traditional areas, include extensive use of 2 × 2 matrices in dealing with ray tracing, polarization, and multiple thin-film interference; three chapters devoted to lasers; a separate chapter on the eye, including laser treatments of the eye; and individual chapters on holography, coherence, fiber optics, interferometry, Fourier optics, nonlinear optics, and Fresnel equations. A final chapter provides a brief introduction to the optical constants of dielectrics and metals. We have attempted to make many of the more specialized chapters independent of the others so that they can be omitted without detriment to the remainder of the book. This should be helpful in designing shorter versions of the course.

Organization of the material in three major parts follows essentially traditional lines. The first part of the book deals with geometrical optics, presented as a limiting form of wave optics. The middle part develops wave optics in detail, and the final part treats topics generally referred to as *modern optics*. In the first part, Chapter 1 presents a brief historical review of the theories of light, including wave, particle, and photon descriptions. In Chapter 2, we describe a variety of common sources and detectors of light, as well as the radiometric and photometric units of measurement that are used throughout the book. In this chapter and the remainder of the text, the rationalized MKS system of units is employed. Chapter 3 reviews the geometrical optics covered by introductory physics courses, deriving the usual reflection and refraction relations for mirrors and lenses. Chapter 4 shows how one can extend paraxial optics to systems of arbitrary complexity through the use of 2 × 2 matrices. Also in this chapter we include an introduction to the ray-tracing techniques that are widely applied using computer programming. Chapter 5 presents a semiquantitative treatment of third-order aberration theory. Chapter 6 discusses the principles of geometrical optics and aberration theory as applied to apertures and to several optical devices: the prism, the camera, the eyepiece, the microscope, and the telescope. The importance of the eye as the final optical instrument in many optical systems is recognized in a separate chapter (7). This chapter explains the functions and the defects of the eye and discusses some of the treatments of these defects that make use of the unique properties of laser light.

The next section of the text introduces wave or physical optics with two chapters (8 and 9) that discuss the wave equations and the superposition of waves. Interference phenomena are then treated in Chapters 10 and 11, the second dealing with both Michelson and Fabry-Perot interferometers in some detail. Although the concept of coherence is handled in general terms in preceding discussions, it receives a more precise and quantitative treatment in Chapter 12. After a brief explanation of Fourier series and the Fourier integral, the chapter deals with both temporal and spatial coherence and presents a quantitative discussion of partial coherence. Chapter 13 presents, as a special application of interference, an introduction to holography, including some current applications.

Chapters 14 and 15 treat the polarization of light. We first give a mathematical presentation using 2 × 2 matrices to represent the electric field vector (Chapter 14), before examining in detail the physical mechanisms responsible for the production of polarized light (Chapter 15). Thus Chapter 14 uses matrices to describe the various modes of polarized light and types of polarizers without reference to the physics of its production. Although the order of these chapters can be reversed, we feel this choice is pedagogically more effective. Diffraction is discussed in the following three chapters (16, 17, 18). Since an adequate treatment of Fraunhofer diffraction is too long for a single chapter, we have included a separate chapter (17) on the diffraction grating and grating instruments immediately following the discussion of multiple-slit diffraction in Chapter 16. Fresnel diffraction is then taken up in Chapter 18.

The final chapters are generally more demanding in mathematical sophistication. Chapter 19 employs 2 × 2 matrices to treat reflectance of multilayer thin films. Chapter 20 derives the Fresnel equations in an examination of reflection from both dielectric and metallic surfaces. The basic elements of a laser and the basic characteristics of laser light are treated in Chapter 21, followed by a rather demanding chapter (22) that describes the principal features of laser beams. The divergence and mode structure of laser beams are dealt with here in a quantitative manner. Chapters 21 and 22 are best taken in sequence, and together with Chapter 23, essentially an essay on laser applications, form a suitable three-chapter unit for a minicourse on lasers. The other chapters in this final part of the book are self-contained in the sense that no particular sequence is required.

Chapter 24 presents a survey of the basic features of optical fibers with special attention given to communication applications. Thus topics of bandwidth, allowed modes, and mechanisms of attenuation and distortion are treated here. Chapter 25 introduces the subject of Fourier optics in a discussion of optical data processing and Fourier-transform spectroscopy. Chapter 26 presents a variety of physical effects under the umbrella of nonlinear optics. The final chapter (27) considers the propagation of a light wave in both dielectric and metallic media and shows how the optical constants arise.

Each of the 27 chapters contains a limited bibliography related to the chapter contents and referred to at times within the text using square brackets. In addition, at the end of the book, we have included a chronological listing of articles related to optics that have appeared in *Scientific American* over the last 40 years or so. It is hoped that this list of excellent articles will prove helpful, especially to the undergraduate student.

This text is intended to be adaptable for either one or two semester sequences. The precise selection of material will depend on the particular goals of both teacher and student. As a rough guide, however, a typical one-semester course might include the basic sequence:

Chapter	1	Nature of Light
	3	Geometrical Optics
	6	Optical Instrumentation
	8	Wave Equations
	9	Superposition of Waves
	10	Interference of Light
	12	Coherence
	13	Holography
	15	Production of Polarized Light
	16	Fraunhofer Diffraction
	18	Fresnel Diffraction
	21	Laser Basics

As a further aid to selection, those sections that could be omitted in abbreviated versions of the course are marked with an asterisk. See the Contents.

We wish to thank the many teachers who have inspired us with an interest in optics and in teaching and the many students who have motivated us to teach with clarity and efficiency. For their very helpful reading of portions of the manuscript for the first edition, we are indebted to Hugo Weichel, James Tucci, Hajime Sakai, Arthur H. Guenther, and Thomas B. Greenslade. For their suggestions for improving the second edition, which we have considered very seriously, we wish to thank the team of reviewers selected by Prentice Hall: Joel Blatt, Florida Institute of Technology; James Boger, Oregon Institute of Technology; Harry Daw, New Mexico State University; Edward Eyler, University of Delaware; and Daniel Wilkins, University of Nebraska. We are grateful to Leno M. Pedrotti for his critical reading of most of the new material added in this second edition. For his review and suggestions in the chapter on the eye, we are also pleased to acknowledge and thank Dr. Michael Pedrotti, O.D. We also wish to thank Judy Lawson for her sketch of Einstein that graces page 1. Finally, we express our gratitude to the editorial and production staff of Prentice Hall. In particular, we are indebted to our acquisitions editors, Holly Hodder for the first edition and Ray Henderson for the second edition, and to our production editor, Kathleen Lafferty, who helped us with both editions of this text.

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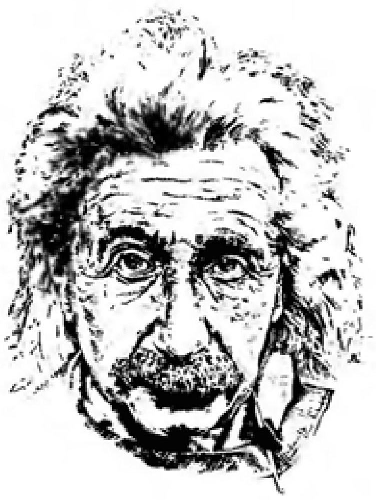

Nature of Light

INTRODUCTION

The evolution in our understanding of the physical nature of light forms one of the most fascinating accounts in the history of science. Since the dawn of modern science in the sixteenth and seventeenth centuries, light has been pictured either as particles or waves—incompatible models—each of which enjoyed a period of prominence among the scientific community. In the twentieth century it became clear that somehow light was both wave and particle, yet it was precisely neither. For some time this perplexing state of affairs, referred to as the *wave-particle duality*, motivated the greatest scientific minds of our age to find a resolution to apparently contradictory models of light. The solution was achieved through the creation of *quantum electrodynamics*, one of the most successful theoretical structures in the annals of physics.

In what follows, we will be content to sketch briefly a few of the high points of this developing understanding.[1] Certain areas of physics once considered to be disciplines apart from optics—electricity and magnetism, and atomic physics—are very much involved in this account. This alone suggests that the resolution achieved also constitutes one of the great unifications in our understanding of the physical world. The final result is that light and subatomic particles, like electrons, are both consid-

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[1] A more in-depth historical account may be found, for example, in [1] (see References at the end of the chapter).

ered to be manifestations of matter or energy under the same set of formal principles.

In the seventeenth century the most prominent advocate of a particle theory of light was Isaac Newton, the same creative giant who had erected a complete science of mechanics and gravity. In his treatise *Optics*, Newton clearly regarded rays of light as streams of very small particles emitted from a source of light and traveling in straight lines. Although Newton often argued forcefully against positing hypotheses that were not derived directly from observation and experiment, here he adopted a particle hypothesis, believing it to be adequately justified by the phenomena. Important in his considerations was the observation that light can cast sharp shadows of objects, in contrast to water and sound waves, which bend around obstacles in their paths. At the same time, Newton was aware of the phenomenon now referred to as *Newton's rings*. Such light patterns are not easily explained by viewing light as a stream of particles traveling in straight lines. Newton maintained his basic particle hypothesis, however, and explained the phenomenon by endowing the particles themselves with what he called "fits of easy reflection and easy transmission," a kind of periodic motion due to the attractive and repulsive forces imposed by material obstacles. Newton's eminence as a scientist was such that his point of view dominated the century that followed his work.

A BRIEF HISTORY

Christian Huygens, a Dutch scientist contemporary with Newton, championed the view (in his *Treatise on Light*) that light is a wave motion, spreading out from a light source in all directions and propagating through an all-pervasive elastic medium called the ether. He was impressed, for example, by the experimental fact that when two beams of light intersected, they emerged unmodified, just as in the case of two water or sound waves. Adopting a wave theory, Huygens was able to derive the laws of reflection and refraction and to explain double refraction in calcite as well.

Within two years of the centenary of the publication of Newton's *Optics*, the Englishman Thomas Young performed a decisive experiment that seemed to demand a wave interpretation, turning the tide of support to the wave theory of light. It was the double-slit experiment, in which an opaque screen with two small, closely spaced openings was illuminated by monochromatic light from a small source. The "shadows" observed formed a complex interference pattern like those produced with water waves.

Victories for the wave theory continued up to the twentieth century. In the mood of scientific confidence that characterized the latter part of the nineteenth century, there was little doubt that light, like most other classical areas of physics, was well understood. We mention a few of the more significant confirmations.

In 1821 Augustin Fresnel published results of his experiments and analysis, which required that light be a transverse wave. On this basis, double refraction in calcite could be understood as a phenomenon involving polarized light. It had been assumed that light waves in an ether were necessarily longitudinal, like sound waves in a fluid, which cannot support transverse vibrations. For each of the two components of polarized light, Fresnel developed the *Fresnel equations*, which give the amplitude of light reflected and transmitted at a plane interface separating two optical media.

Working in the field of electricity and magnetism, James Clerk Maxwell synthesized known principles in his set of four *Maxwell equations*. The equations yielded a prediction for the speed of an electromagnetic wave in the ether that turned out to be the measured speed of light, suggesting its electromagnetic character. From then on, light was viewed as a particular region of the electromagnetic spectrum of radiation. The experiment (1887) of Albert Michelson and Edward Morley, which attempted to detect optically the earth's motion through the ether, and the special theory of relativity (1905) of Albert Einstein were of monumental importance. Together they led inevitably to the conclusion that the assumption of an ether was superfluous. The problems associated with transverse vibrations of a wave in a fluid thus vanished.

If the nineteenth century served to place the wave theory of light on a firm foundation, this foundation was to crumble as the century came to an end. The wave-particle controversy was resumed with vigor. Again, we mention only briefly some of the key events along the way. Difficulties in the wave theory seemed to show up in situations that involved the interaction of light with matter. In 1900, at the very dawn of the twentieth century, Max Planck announced at a meeting of the German Physical Society that he was able to derive the correct blackbody radiation spectrum only by making the curious assumption that atoms emitted light in discrete energy chunks rather than in a continuous manner. Thus *quanta* and *quantum mechanics* were born. According to Planck, the energy E of a quantum of electromagnetic radiation is proportional to the frequency of the radiation, ν,

$$E = h\nu \tag{1-1}$$

where the constant of proportionality, *Planck's constant*, has the very small value of 6.63×10^{-34} J-s. Five years later, in the same year that he published his theory of special relativity, Albert Einstein offered an explanation of the photoelectric effect, the emission of electrons from a metal surface when irradiated with light. Central to his explanation was the conception of light as a stream of photons whose energy is related to frequency by Planck's equation (1-1). Then in 1913 the Danish physicist Niels Bohr once more incorporated the quantum of radiation in his explanation of the emission and absorption processes of the hydrogen atom, providing a physical basis for understanding the hydrogen spectrum. Again in 1922, the photon model of light came to the rescue for Arthur Compton, who explained the scattering of X-rays from electrons as particlelike collisions between photons and electrons in which both energy and momentum were conserved.

All such victories for the photon or particle model of light indicated that light could be treated as a particular kind of matter, possessing both energy and momentum. It was Luis de Broglie who saw the other side of the picture. In 1924 he published his speculations that subatomic particles are endowed with wave properties. He suggested, in fact, that a particle with momentum p had an associated wavelength of

$$\lambda = \frac{h}{p} \tag{1-2}$$

where h was, again, Planck's constant. Experimental confirmation of de Broglie's hypothesis appeared during the years 1927–1928, when Clinton Davisson and Lester Germer in the United States and Sir George Thomson in England performed experiments that could only be interpreted as the diffraction of a beam of electrons.

Thus the wave-particle duality came full circle. Light behaved like waves in its propagation and in the phenomena of interference and diffraction; it could, however, also behave as particles in its interaction with matter, as in the photoelectric effect. On the other hand, electrons usually behaved like particles, as observed in the point-like scintillations of a phosphor exposed to a beam of electrons; in other situations they were found to behave like waves, as in the diffraction produced by an electron microscope.

Photons and electrons that behaved both as particles and as waves seemed at first an impossible contradiction, since particles and waves are very different entities indeed. Gradually it became clear, to a large extent through the reflections of Niels Bohr and especially in his *principle of complementarity*, that photons and electrons were neither waves nor particles, but something more complex than either.

In attempting to explain physical phenomena, it is natural that we appeal to well-known physical models like waves and particles. As it turns out, however, the full intelligibility of a photon or an electron is not exhausted by either model. In certain situations, wavelike attributes may predominate; in other situations, particlelike attributes stand out. We can appeal to no simpler physical model that is adequate to handle all cases.

Quantum mechanics, or wave mechanics, as it is often called, deals with all particles more or less localized in space, and so describes both light and matter. Combined with special relativity, the momentum p, wavelength λ, and speed v for both material particles and photons are given by the same general equations:

$$p = \frac{\sqrt{E^2 - m^2c^4}}{c} \tag{1-3}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E^2 - m^2c^4}} \tag{1-4}$$

$$v = \frac{pc^2}{E} = c\sqrt{1 - \frac{m^2c^4}{E^2}} \tag{1-5}$$

In these equations, m is the *rest mass* and E is the total energy, the sum of rest-mass energy mc^2 and kinetic energy, that is, the work done to accelerate the particle from rest to its measured speed. The relativistic mass is given by γm, where γ is the ratio $1/\sqrt{1 - (v/c)^2}$. The proper expression for kinetic energy is no longer simply $\frac{1}{2}mv^2$, but $mc^2(\gamma - 1)$. The relativistic expression for kinetic energy approaches $\frac{1}{2}mv^2$ for $v \ll c$.[2]

A crucial difference between particles like electrons and neutrons and particles like photons is that the latter have zero rest mass. Equations (1-3) to (1-5) then take the simpler forms for photons:

$$p = \frac{E}{c} \tag{1-6}$$

$$\lambda = \frac{h}{p} = \frac{hc}{E} \tag{1-7}$$

$$v = \frac{pc^2}{E} = c \tag{1-8}$$

Thus, while nonzero rest-mass particles like electrons have a limiting speed of c, Eq. (1-8) shows that zero rest-mass particles like photons must travel with the constant speed c. The energy of a photon is not a function of its speed but of its frequency, as expressed in Eq. (1-1) or in Eqs. (1-6) and (1-7), taken together. Notice that for a photon, because of its zero rest mass, there is no distinction between its total energy and its kinetic energy. The following example illustrates the preceding equations.

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[2] This discussion is not meant to be a condensed tutorial on relativistic mechanics, but, with the help of Eqs. (1-3) to (1-8), a summary of some basic relations that unify particles of matter and light.

Example

An electron is accelerated to a kinetic energy of 2.5 MeV. Determine its relativistic momentum, de Broglie wavelength, and speed. Also determine the same properties for a photon having the same energy as the electron.

Solution The electron's total energy E must be the sum of its rest mass energy and its kinetic energy, E_k:

$$E = mc^2 + E_k = 0.511 \text{ MeV} + 2.5 \text{ MeV} = 3.011 \text{ MeV}$$

or

$$E = 3.011 \times 10^6 \text{ eV} \times (1.602 \times 10^{-19} \text{ J/eV}) = 4.82 \times 10^{-13} \text{ J}$$

The other quantities are then calculated in order. From Eq. (1-3):

$$p = 1.58 \times 10^{-21} \text{ kg-m/s}$$

From Eq. (1-4):

$$\lambda = 41.8 \times 10^{-12} \text{ m} = 41.8 \text{ pm}$$

From Eq. (1-5):

$$v = 2.95 \times 10^8 \text{ m/s}$$

For the photon, with $m = 0$, we get instead From Eq. (1-6):

$$p = 1.61 \times 10^{-21} \text{ kg-m/s}$$

From Eq. (1-7):

$$\lambda = 0.412 \text{ pm}$$

From Eq. (1-8):

$$v = c = 3.00 \times 10^8 \text{ m/s}$$

Another important distinction between electrons and photons is that electrons obey Fermi statistics whereas photons obey Bose statistics. A consequence of Fermi statistics is the restriction that no two electrons in the same interacting system be in the same *state*, that is, have precisely the same physical properties. Bose statistics impose no such prohibition, so that identical photons with the same energy and momentum can occur together in large numbers. Because light beams can possess so many similar photons in proximity, the granular structure of the beam is not ordinarily experienced, and the beam can be adequately represented by a continuous electromagnetic wave. From this point of view, electromagnetic fields appear as a special manifestation of photons.

A profound consequence of the wave nature of particles is embodied in the Heisenberg principle of indeterminacy. As a result of this principle, particles do not obey deterministic laws of motion. Rather, the theory predicts only probabilities. Wave functions are associated with the particles through the fundamental wave equation of quantum mechanics. The wave amplitudes, or, better, the square of the wave amplitudes assigned to these particles, provide a means of expressing the probability that a particle will be found within a region of space during an interval of time. Thus the *irradiance* (power/area) of these waves at some intercepting surface, also proportional to the square of the wave amplitudes, provides a measure of this probability. When large numbers of particles are involved, probabilities approach certainties, so that the irradiance E_e of light at a location is proportional to the number of photons passing through the location per second.

$$n \text{ (photons/m}^2\text{-s)} = \frac{E_e}{h\nu} \tag{1-9}$$

In this way, the interference and diffraction patterns previously explained by waves can be interpreted as manifestations of particles. Particle wave amplitudes predict the probabilities of their locations in the same patterns.

In the theory called quantum electrodynamics, which combines the principles of quantum mechanics with those of special relativity, photons are assumed to interact only with charges. An electron, for example, is capable of both absorbing and emitting a photon, with a probability that is proportional to the square of the charge. There is no conservation law for photons as there is for the charge associated with particles. In this theory the wave-particle duality becomes reconciled. Essential distinctions between photons and electrons are removed. Both are considered subject to the same general principles. Through this unification, light is viewed as basically just another form of matter. Nevertheless, the complementary aspects of particle and wave descriptions of light remain, justifying our use of one or the other description when appropriate. The wave description of light will be found adequate to describe most of the optical phenomena treated in this text.

PROBLEMS

1-1. Calculate the de Broglie wavelength of (a) a golf ball of mass 50 g moving at 20 m/s and (b) an electron with kinetic energy of 10 eV.

1-2. The threshold of sensitivity of the human eye is about 100 photons per second. The eye is most sensitive at a wavelength of around 550 nm. For this wavelength determine the threshold in watts of power.

1-3. What is the energy, in electron volts, of light photons at the ends of the visible spectrum, that is, at wavelengths of 400 and 700 nm?

1-4. Determine the wavelength and momentum of a photon whose energy equals the rest-mass energy of an electron.

1-5. Show that the rest-mass energy of an electron is 0.511 MeV.

1-6. Show that the relativistic momentum of an electron, accelerated through a potential difference of 1 million volts, can be conveniently expressed as 1.422 MeV/c, where c is the speed of light.

1-7. Show that the wavelength of a photon, measured in angstroms, can be found from its energy, measured in electron volts, by the convenient relation,

$$\lambda\,(\text{Å}) = \frac{12{,}400}{E(\text{eV})}$$

1-8. Show that the relativistic kinetic energy,

$$E_k = mc^2(\gamma - 1)$$

reduces to the classical expression, $\frac{1}{2}mv^2$, when $v \ll c$.

1-9. A proton is accelerated to a kinetic energy of 2 billion electron volts (2 BeV). Find (a) its momentum; (b) its de Broglie wavelength; (c) the wavelength of a photon with the same total energy.

1-10. Solar radiation is incident at the earth's surface at an average of 1353 W/m^2 on a surface normal to the rays. For a mean wavelength of 550 nm, calculate the number of photons falling on 1 cm^2 of the surface each second.

1-11. Two parallel beams of electromagnetic radiation with different wavelengths deliver the same power to equivalent surface areas normal to the beams. Show that the numbers of photons striking the surfaces per second for the two beams are in the same ratio as their wavelengths.